

Exercise 43

Let $f(x) = \frac{1}{x}$. Find the number b such that the average rate of change of f on the interval $(2, b)$ is $-\frac{1}{10}$.

Solution

Set $-1/10$ equal to the average rate of change from $x = 2$ to $x = b$, and solve the resulting equation for b .

$$\begin{aligned} -\frac{1}{10} &= \frac{f(b) - f(2)}{b - 2} \\ &= \frac{\frac{1}{b} - \frac{1}{2}}{b - 2} \\ &= \frac{\frac{2}{2b} - \frac{b}{2b}}{b - 2} \\ &= \frac{\frac{2-b}{2b}}{b - 2} \\ &= \frac{2 - b}{2b(b - 2)} \\ &= \frac{-(b - 2)}{2b(b - 2)} \\ &= -\frac{1}{2b} \end{aligned}$$

Therefore, $10 = 2b$, which means $b = 5$.